Ancient Jewish Mathematical Astronomy

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Communicated by B. L. VAN DER WAERDEN

1. Introduction

The new moon was proclaimed in Israel on the basis of observation until the mid-fourth century C.E.[1] In his treatise *Sanctification of the New Moon*,[2] (completed *circa* 1178), Rabbi MOSHE BEN MAIMON (MAIMONIDES) asserts (1:6, 6:1, 11:1)[3] that each month, the court calculated astronomically whether the new crescent would be visible on the evening preceding the 30th day of the month; only when visibility was deemed possible, would the court be in session on the 30th day to accept witnesses. Now a lunar visibility theory is no trifling matter: Greek mathematical astronomy, which reached its culmination in the *Almagest* of PTOLEMY, shows no trace of such a theory[4]. It is to be expected, then, that some scholars would find MAIMONIDES' assertion startling[5]. Their skepticism is fueled, no doubt, by the apparent lack of evidence—in talmudic and midrashic literature—of the existence of a mathematical lunar theory. In this paper, I will show that it is very plausible, perhaps even probable, that the talmudic calendar council (*sod ha'ibbur*) possessed a lunar theory whose visibility component ultimately reached the hands of MAIMONIDES.

To advance our thesis, I will present and analyze both the Maimonidean

[1] See note 66 below. "C.E." stands for Common Era, of which the present year is 1987.

[2] The best edition of the Code of MAIMONIDES is *Mishneh Torah*, Vol. 2: *Sefer zĕmanim*, Shabse Frankel, Jerusalem 5735 (1975); it contains variant readings from manuscripts and early editions. The English translation by S. GANDZ was published as *The Code of Maimonidees, Sanctification of the new moon*, Yale Univ. Press, New Haven, 1956; it contains an introduction by J. OBERMANN and an astronomical commentary by O. NEUGEBAUER. For the quotations in this paper I found it preferable to do my own translating from the Hebrew.

[3] All references to MAIMONIDES are to chapter and section of *Sanctification*.

[4] See O. NEUGEBAUER, *A history of ancient mathematical astronomy*, Springer-Verlag, Berlin Heidelberg New York, 1975, p. 829.

[5] OBERMANN[2], p. xvii.

evidence, and the midrashic-talmudic evidence. First of all, it will be demonstrated that MAIMONIDES' visibility theory itself (Ch. 17) points to a Jewish source[6] dating back to the period when the new moon was determined by observation. But the main task of this article will be to uncover and analyze Hebrew mathematical astronomy from the talmudic period (1st through 5th centuries), most of which has been overlooked until now.

The most direct link is a midrashic visibility parameter which, as I will show in Section 2, is in complete accord with MAIMONIDES' visibility criterion; but the main extant body of ancient Jewish mathematical astronomy is found in the first half of *Baraitha diShmuel* (BdS)[7], which probably dates from the talmudic period, as we shall see in Section 3. Although first published in 1861, its importance for the history of Jewish astronomy has not been noticed until now. Its Babylonian-type arithmetical methods, which are couched in somewhat obscure verbal form, will be elucidated in Section 3; only since the publication, in the last few decades, of many texts in Babylonian and ancient Indian astronomy, could the astronomy of BdS be properly analyzed and appreciated. In Sections 4 and 5, I will discuss additional midrashic parameters, and evaluate the pertinent talmudic evidence. The picture that will emerge is that Hebrew mathematical astronomy was practiced in the talmudic period, and that significant traces of it have remained in spite of the inherent secrecy connected with the *sod haʿibbur*.

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2. The visibility criterion

The lunar theory which MAIMONIDES presents in Chapters 11–16 and Chapter 19 is based on the exposition of Ptolemaic astronomy by Muslim astronomers, notably AL-BATTANI. This was demonstrated by NEUGEBAUER[8] and is alluded to by MAIMONIDES himself in 17: 25. But the question of his sources for Chapter 17, which gives the actual procedure for determining visibility or invisibility, is a different matter, as we shall see below.

Of special interest in Chapter 17 is the visibility criterion, which can be summarized as follows: let λ_1 denote the true elongation for the evening in question, and let b be the "arc of vision"[8a], *i.e.*, the length of the equator arc which sets between sunset and moonset of that evening. Then the moon will surely be invisible if $\lambda_1 \leqq 9°$, and surely visible if $\lambda_1 > 15°$, provided the moon is in the spring semicircle. For the autumn semicircle, the corresponding limits are given as 10° and 24° (17: 3, 4). When λ_1 is between the above limits, then we have

[6] As we shall see below, the basic direction has been pointed out by NEUGEBAUER.

[7] *Baraitha diShmuel*, Saloniki 5621 (1861).

[8] O. NEUGEBAUER, "The astronomy of Maimonides", *Hebrew Union College annual* xxii (1949), 322–363, pp. 334–349, and his commentary to *Sanctification*², pp. 123–136.

[8a] "Arc of vision" is GANDZ's translation of MAIMONIDES' *qesheth rĕiyyah*. It should not be confused with the term *arcus visionis*.

visibility if and only if the two conditions, $b > 9°$ and $\lambda_1 + b \geqq 22°$, are both satisfied (17:15–21).

Due to MAIMONIDES' methodology in Chapter 17—including his roundabout method of calculating the arc of vision, and his crude estimate of the quota of geographical latitude—NEUGEBAUER[9] is of the opinion that "Maimonides depends in the whole section of visibility on much more primitive sources than in the computation of the position of the moon." Furthermore, none of the Muslim astronomers had given a criterion of the form $\lambda_1 + b \geqq c$; therefore, NEUGEBAUER[10] concludes that it "seems plausible to assume that Maimonides follows in the problem of visibility a Jewish tradition, uninfluenced by Arabic methods."

Let us now turn to MAIMONIDES' own words. In explaining his decision to record the methods of calculation for visibility, he writes (11:3):

> Moreover, we have—concerning these rules—traditions handed down by the Sages, and proofs, which are not written in commonly known books.

From the context, it would seem that the Sages referred to are Jewish sages.

If we are indeed dealing with a Jewish, non-Arabic visibility theory, then the most logical time-period for the origin of this tradition would be the talmudic era, when the new month was proclaimed on the basis of observation; only then did the rabbinical court have to contend, monthly, with the practical problem of visibility. Consequently, the very visibility theory recorded by MAIMONIDES would tend to support his assertion of astronomical calculation on the part of the court.

It should be added that the borderline cases of the Babylonian ephemerides, which deal with lunar visibility[11], seem to suggest that a criterion of the form $\lambda_1 + b \geqq c$ was used[12]. Thus, MAIMONIDES' criterion would fit into the framework of Babylonian astronomy which was contemporary with Israel's Second Temple period.[13]

The special court which dealt with calendrical questions was called *sod ha'ibbur*, literally, "the secret council of intercalation", and membership was restricted to a small, select group of talmudic sages[14]. Thus, one should not expect its deliberations and methods to be disseminated to the talmudic rabbinical community

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[9] The astronomy of MAIMONIDES[8], p. 356.

[10] *ibid.*, p. 360.

[11] O. NEUGEBAUER, *Astronomical cuneiform texts*, Lund Humphries, London 1955, texts 5, 7, 12, 18 (System A), texts 100, 101, 102, 120, 122 (System B). Their dates range from 206 B.C.E. to 48 B.C.E.

[12] NEUGEBAUER[11], pp. 67, 84.

[13] The Second Temple was destroyed in 70 C.E. In making the comparison with Babylonian astronomy we refer only to the criterion *per se*: $\lambda_1 + b \geqq 22°, b > 9°$, *etc.*, but not to all the calculational details in Chapter 17. The setting times for the zodiacal signs, which are reflected in 17:12, are rounded off from trigonometrical computation and not based on a simple arithmetical scheme such as one finds in Systems A and B (see NEUGEBAUER[4], p. 368). Also, there does not seem to be any evidence in Babylonian astronomy of a correction for parallax (17:5–9).

[14] See, *e.g.*, the statement of Rabbi EL'AZAR (second half of 3rd century C.E.) in Bab. Talmud Kĕthubboth 112a.

at large[15]; nevertheless, if the court possessed and used a lunar theory, one would expect to find some traces of this fact in talmudic and midrashic literature. As mentioned above, we will analyze the astronomy-related material in that literature with an eye to uncovering those traces and remnants.

An important parameter for our purposes is the least value of the elongation λ_1 for which visibility is possible. A careful inspection of MAIMONIDES' procedure in Chapter 17 reveals that the inequality $\lambda_1 + b \geqq 22°$ implies $\lambda_1 \geqq 9° 8' 9''$. (This minimum of λ_1 is attained when the true position of the moon is near the end of Gemini.) Thus, when he gave 9° as the upper limit of sure invisibility (17:3), it was the result of rounding down to an integral number of degrees.

A related parameter is found in the midrash called *Pirqey dĕRabbi Eliʿezer*[16] (PdRE), Chapter 7:

> The eye is unable to see the moon until eight large hours, whether before the lunar conjunction or after the lunar conjunction.

That these "large hours" refer to double hours[17], is clear from the beginning of the 5th Chapter of BdS.

According to the lunar theory in MAIMONIDES, an elongation of 9° 8′ 9″ can be attained as early as $15^h 58^m$ after conjunction. (This is the case if the moon is at perigee, and the sun at apogee, when the elongation is 9° 8′ 9″. Since the double elongation at that time is 18° 16′, the moon's *mean* anomaly must have been 178° (see MAIMONIDES 15 : 3). MAIMONIDES' table for the quota of solar anomaly (13 : 4) can be used to compute the sun's true position $15^h 58^m$ earlier, which is the time of the true conjunction. But his table for the quota of lunar anomaly (15 : 6) was constructed for the evening of first visibility, and is based on a mean elongation of 15°, while the mean elongation at the time of our true conjunction was about 1°; thus, in order to compute correctly the moon's true position at conjunction, we must revert to AL-BATTANI'S tables.[18]) Thus, the 16 hour minimum[19] of PdRE is in line with the visibility criterion recorded by MAIMONIDES.

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15 Compare Rabbenu ḤANANEL'S commentary to Rosh Hashanah 20a: "It was called *sod haʿibbur* because it was revealed only to those who were invited to deliberate." See also note 68 below.

16 The best edition is *Pirqey Rabbi Eliʿezer*, Warsaw 5612 (1852), with the commentary of Rabbi DAVID LURIA, which quotes many valuable parallels and citations. The English edition is *Pirkê de Rabbi Eliezer*, transl. G. FRIEDLANDER, London 1916. The reader should be warned that FRIEDLANDER'S Introduction, insofar as it deals with the question of date and provenance, is outdated.

17 In some manuscripts of PdRE it is written explicitly "large hours which are two hours each." (Three manuscripts were published by M. HIGGER, "Pirqey Rabbi Eliʿezer", *Horeb* viii (5704/1944), 82–119. FRIEDLANDER'S translation[16] is based on yet another manuscript.)

18 AL-BATTANI, *Opus astronomicum*, ed. C. A. NALLINO, Part II, Rome 1907, pp. 78–83, column "Aequatio simplex Lunae".

19 It is interesting to compare this with modern estimates. F. X. KUGLER (*Sternkunde und Sterndienst in Babel* II, Munster, Aschendorff 1909–1924, p. 434) has 15.5^h, while C. SCHOCH (S. LANGDON, J. K. FOTHERINGHAM, & C. SCHOCH, *The Venus tablets of Ammizaduga*, Oxford 1928, p. 97) gives 16.5^h. (Quoted from NEUGEBAUER[11], p. 67.)

PdRE in its present form was redacted in the Land of Israel during the Umayyad dynasty (660–750), certainly before the fall of that dynasty[20]; most of the material embedded in PdRE is much older. Islamic astronomy in the fertile crescent emerged at the Abbasid court in Baghdad in the latter half of the 8th century[21], so that any astronomy found in PdRE must be pre-Arabic.

3. Baraitha diShmuel

BdS is our main source of ancient Hebrew astronomy; but before analyzing its astronomy, we must deal with some textual questions. Beginning with R. SHABBĔTHAI DONOLO (born 913), many medieval Jewish writers cited a work which they called Baraitha diShmuel[22] and which they attributed to the Babylonian talmudic sage SHĔMUEL (died 254 C.E.). In 1861, NATHAN ʿAMRAM published (from an old manuscript) an astronomical-astrological text which he called Baraitha diShmuel, containing nine chapters. Those parts of ʿAMRAM's text which appear as medieval quotations from BdS, are found only in the first four and a half chapters. G. B. SARFATTI[23] showed that an analysis of the text leads to the conclusion that the second half of ʿAMRAM's book (beginning with "Twelve *mazzaloth* ..." in the middle of Chapter 5), is a separate—and later—work. Accordingly, I will label the first four and a half chapters as "BdS1", and the rest as "BdS2". In this article, we are concerned only with BdS1.

We must still clarify the question of the first half of Chapter 5. It begins "In the year 4536 [Anno Mundi = 776 C.E.] the sun and moon ... became equal ...". It then gives the procedure for finding the lunar *molad* (conjunction), using 4536 as its epoch. For the next procedure, the computation of the *tĕqufah* (equinox), the epoch used is the creation of the world.

Now when R. AVRAHAM BAR ḤIYYA quotes from BdS in his *Sefer haʿibbur*[24]

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[20] See the article "Pirkei de-Rabbi Eliezer" by M. D. HERR in *Encyclopaedia Judaica*, Vol. 13, Jerusalem 1971, col. 558–560, and his Bibliography.

[21] D. PINGREE, "The Greek influence on early Islamic mathematical astronomy", *Journal of the American Oriental Society* xciii (1973), 32–43, pp. 36–37, pointed out that the earliest known astronomical texts in Arabic were written in Sind and Afghanistan within the last 15 years of the Ummayad dynasty; but these could hardly have influenced the redaction of PdRE.

[22] For references to lists of those citations, see G. B. SARFATTI, "An introduction to 'Barayta De-Mazzalot'", *Bar Ilan Univ. annual* iii (1965), 56–82 (in Hebrew), p. 72, note 51, pp. 76–77. To those lists one can add: a) Rabbenu Ḥananel to Pĕsaḥim 94a (which is itself a verbatim quotation from a responsum of Rav SHĔRIRA GAON (906–1006) and Rav HAI GAON (939–1038), see B. M. LEWIN, *Otzar ha-Geonim*, Vol. 3, Jerusalem 5740 (1980), p. 88). b) Tosĕfoth haRosh to Rosh Hashanah 8a (and the reading in Tosafoth *ad loc.* should be corrected accordingly). c) *Shiṭṭah Mĕqubbeṣeth* Kĕthubboth 111a. d) See note 31 below.

[23] SARFATTI[22], p. 72–78.

[24] AVRAHAM BAR ḤIYYA, *Sefer haʿibbur*, ed. H. FILIPOWSKI, London 1851, p. 36 (from ms. Paris). For a better reading (from ms. Oxford), see FILIPOWSKI's introduction, p. xiii.

(*circa* 1121), he omits the opening sentences which introduce the epoch of 4536, and quotes the procedure for computing the *molad*, but the epoch he quotes is the creation epoch. Similarly, we find that R. AVRAHAM IBN ʿEZRA, in his commentary to Exodus 12:2 (*circa* 1150) also quotes the Baraitha's instructions for finding the *molad* with the creation epoch.

The most plausible conclusion one can draw from the above citations is as follows: (1) The original version of BdS1 did not contain the epoch of 4536, and Chapter 5 began with the words "*haroṣeh ledaʿ*" ("he who wishes to know"). (2) The original version used the creation epoch for the procedures of computing both the *molad* and the *tĕqufah*. This is the version that reached the hands of BAR ḤIYYA and IBN ʿEZRA. (3) The opening sentences which introduce the epoch 4536 A.M. were interpolated into a BdS manuscript (probably soon after 776 C.E.), and the procedure for finding the *molad* was rewritten using the new epoch, but the *tĕqufah* procedure was left untouched. It was a descendent of the interpolated manuscript that finally reached NATHAN ʿAMRAM in 1861.

Thus, BdS1 was written earlier than 776 C.E. After discussing the astronomy of BdS1, we will be in a position to conclude (in Subsection E) that it probably dates from the talmudic period.

A. Eclipse theory

In the latter part of Chapter 1 of BdS1 after discussing the nodal line (*tĕli*), and the "head" and "tail" of the *tĕli* (ascending and descending node), it says:

> The head moves left-handedly[25], the tail moves backwards, the luminaries and planets move forwards. If the luminaries meet with the nodal line ... one at its head, and the other at its tail, or both at its head, or both at its tail, whether before it or after it, whether both of them are on one side, or the two of them are on two sides, 'sun and moon darken' (Joel 4:15).

After this qualitative discussion of the cause of eclipses, BdS1 continues (beginning of Ch. 2):

> New moon and the day after, they are both at the head or at the tail. 15th and 16th, one is at the head and one at the tail. The day of the *molad* for the sun, its night is the middle[26] of the lunar month. And one must know which zodiacal sign the node is in, and when the sun enters, and in how many [degrees lon-

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[25] Read *ʾṭr*, according to the testimony of Rabbi YIṢḤAQ IBN GHAYYAT (1030–1089) in his liturgical poem "'Asitha arbaʿah adanim" (printed in *Sifthei rĕnanoth*, Prayers and seliḥoth according to the custom of Tripoli and Gerba, Gerba 5707 (1947), part II, p. 19b), into which he wove several sentences from BdS. (The printed edition of BdS has the unintelligible *ʾwšr*.) "Left-handed" denotes retrograde motion in the sense that left-handedness is the opposite of the usual situation. The pair *iṭṭer-hafukh* is but one example of the many synonym-pairs in BdS.

[26] Read *bḥṣy* instead of *wḥṣy*.

gitude] was the moon born, in which day of the *tĕqufah* was it born, and in which day of the week. And for the *molad* calculate for each day one hour which is more according to the calculation (?). For the hour and the part, they are the eclipse for the sun in the day of the *tĕqufah*, and for the moon in the middle of the month.
And they [sun and moon] are the ones which are close to the nodal line, at its head or its tail, and the beginner begins and ends. And how close they are: 12; 21 degrees for the sun, 12; 21 for the moon. And how much the node begins or ends in that sign of the zodiac. 20 33 64;30 205;9 for the sun; 1;40 35;9 for the moon. And what is the magnitude of the eclipse, beginning or ending, a sixth from (?) a sixth, a third from a third, two thirds it departs (?) into the zodiacal sign, 'and I will cause the sun to set at noon, and I will darken the earth' (Amos 8 : 9).

While not all the details are clear[27], we have here a summary of a procedure of mathematical astronomy in which the positions of the sun, moon, and nodes are to be computed for the time of conjunction or opposition, in order to ascertain whether an eclipse will occur, and what its magnitude will be.

Of special interest are the ecliptic limits, *i.e.*, the maximum distance between the sun (or moon, measured along the ecliptic) and the nodes, for which an eclipse is possible. The same limit 12° 21′ is given for both solar and lunar eclipses. It would seem that the sentence "And how much the node begins or ends in that sign of the zodiac" should immediately follow the phrase "one must know which zodiacal sign the node is in"; in other words, one must compute how many degrees the node is from either end of the sign. I cannot explain the sequence of numbers from 20 to 35;9. The last sentence seems to list the eclipse magnitudes 1/6, 1/3, 2/3, total.

This passage on eclipses shows that in BdS1 we are dealing with mathematical, computational astronomy. The following passages will show that its methods are Babylonian-type arithmetical methods.

B. Oblique ascensions

In Chapter 3 of BdS1, after a qualitative discussion of the sun's amplitude at rising and setting, and length of daylight, as a function of the seasons, it states "Their ascensions are not equal", and proceeds to give the oblique ascensions[28]

[27] A few remarks concerning the astronomical terminology in the above passage. The novel phrase "*molad* for the sun" is used to denote the day preceding the night of opposition (when a lunar eclipse is possible). This is parallel to the usual phrase "*molad* of the moon" which refers to conjunction (when a solar eclipse is possible). The moon's being "born" (*nolĕdah*) refers, of course, to the conjunction. The first time the word *tĕqufah* is used, it has the usual meaning of a quarter of the year between equinox and solstice. But the second time ("for the sun in the day of the *tĕqufah*") it refers to the conjunction.

[28] The oblique ascension of an ecliptic arc is defined as the length of the equatorial arc which rises simultaneously with it.

of the zodiacal signs, according to the following scheme:

Aries	Pisces	20°
Taurus	Aquarius	24°
Gemini	Capricorn	28°
Cancer	Sagittarius	32°
Leo	Scorpio	36°
Virgo	Libra	40°

This is precisely the oblique ascension scheme of Babylonian System A[29]. BdS1 then goes on to demonstrate the dependence of the length of daylight on the ascensions: When the sun enters Aries, the semicircle of the ecliptic which rises during the day is

> from the beginning of Aries to the beginning of Libra, [having an oblique ascension of] 180 in the day, 180 in the night [semicircle; the equator rises at the rate of] 15 degrees per [seasonal] hour during the day, and likewise at night. [When the daytime semicircle runs] from the beginning of Cancer to the beginning of Capricorn ... 18 degrees[30] per [seasonal] hour by day, and 12 degrees per hour at night ... [when the semicircle runs] from the beginning of Capricorn to the beginning of Cancer, it is reversed: three fifths at night and two fifths by day[31].

C. Lunar phases

Another example of a Babylonian-type linear zigzag function is to be found in Chapter 2 of BdS1, where the illuminated portion of the moon's disc is given for various elongations, as summarized by the following table:

Elongation	0°	30°	60°	90°	120°	180°
Illuminated fraction[32]	0	1/6	1/3	1/2	2/3	1

The observed (accurate) fractions at conjunction, quadrature, and opposition are 0, 1/2, and 1, and the rest is filled in linearly[33].

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[29] NEUGEBAUER[4], p. 368.

[30] Because the day semicircle has an ascension of 216° according to the above scheme.

[31] An explicit statement of the Babylonian ratio 3 : 2 for longest night to shortest day (which is, of course, a corollary of the ascension scheme). In a tosafist (12^{th}–13^{th} centuries) commentary to Genesis 1: 14, BdS is quoted as saying "In the summer solstice the day is 14 hours and two fifths, and the night 9 hours and three fifths ... in the winter solstice the day is 9 hours and three fifths, and the night 14 and two fifths." This commentary is printed in *Tosafot hashalem, Commentary on the Bible*, ed. R. JACOB GELLIS, Vol. I, Jerusalem 5742/1982, pp. 46–47.

[32] For the elongation of 120°, read, for the fraction, *šlyšyym* instead of *šlšym.*

[33] None of the known cuneiform texts lists illuminated fractions other than at syzygies and quadratures. However, the apocryphal Book of Enoch has the same linear zigzag function for the moon's illumination (with the day of the month as its argument), both in the Aramaic fragments found near the Dead Sea at Qumran (see J. T. MILIK, *The*

D. Shadow formula

Finally, we come to the gnomon text in BdS1, Chapter 3. The gnomon is referred to as *ṣel hamaʿaloth*, a biblical term (Isaiah 38 : 8), and its height is given as 12 fingers. The phrase "southern shadow 72" means the noon shadow, when the sun is at its southernmost point. "72" no doubt refers to the total of the twelve monthly noon shadows, which are given according to the following scheme:

	Leo	Virgo	Libra	Scorpio	Sagittarius[34]		
Cancer						Capricorn	
	Gemini	Taurus	Aries	Pisces	Aquarius		(1)
0	2	4	6	8	10	12	

The procedure given in BdS1 for finding the hour of day from the shadow can be summarized by the following formula:

$$t = \frac{72}{s + 12 - s_0}, \tag{2}$$

where t denotes the number of seasonal hours which elapsed since sunrise (in the morning) or which remain until sunset (in the afternoon); s stands for the length of the shadow (in fingers), and s_0 is the shadow at noon. We are told to compute s_0 "according to the month and the day of the month", in other words, by linear interpolation from table (1) according to the day of the zodiacal month. (In BdS1, the cardinal points of the solar year, and therefore of the above noon shadows, are located at the zero points of their respective signs.)

An equivalent[35] of formula (2) appears in the *Yavanajātaka* (79 : 32), a Sanskrit astrological work written by SPHUJIDHVAJA in 270 C.E.[36] Now the *Yavanajātaka* is a versification of a lost Sanskrit prose translation of a Greek astrological text (also lost). The prose Sanskrit translation was made by a certain YAVANEŚVARA in 150 C.E. D. PINGREE[37] has shown that the astronomy of the *Yavanajātaka* is based on Babylonian arithmetical methods (transmitted by the Greeks.) Formula (2) (and SPHUJIDHVAJA's equivalent) recalls the formula $t = c/s$ upon which the cuneiform Mul Apin shadow table[38] is based. (c is constant for any given day,

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Books of Enoch, Oxford 1976, pp. 278–284, 292), and in the Ethiopic Book of Enoch (*The 'astronomical' chapters of the Ethiopic Book of Enoch*, transl. & comm. O. NEUGEBAUER, Copenhagen 1981, 73:4–8, 78:6–8). In other instances, the Book of Enoch seems to have borrowed from Babylonian astronomy (see NEUGEBAUER, *op. cit.*, pp. 11, 12).

[34] For the Sagittarius shadow, read *ʿśrh* instead of *ʿświyh*.

[35] $t = (1/2)\, dg/(s + g - s_0)$, where g is the height of the gnomon, and d is the length of daylight (in the same units as t).

[36] SPHUJIDHVAJA, *The Yavanajatāka of Sphujidhvaja*, ed. transl. & comm. D. PINGREE, Harvard Univ. Press, Cambridge, Mass. and London, 1978. Another equivalent formula is given in the *Pañcasiddhāntikā* IV, 48, (VARĀHAMIHIRA, *Pañcasiddhāntikā*, ed. transl. & comm. O. NEUGEBAUER & D. PINGREE, 2 vols., Copenhagen 1970–1971), which was written in the mid-sixth century C.E.

[37] *ibid.*

[38] See NEUGEBAUER[4], p. 554.

and varies with the seasons.) The idea common to both is that time elapsed is inversely proportional to the (modified) shadow length.

The noon shadows in (1) are intriguing; they do not fit the latitude of Babylonia and the Land of Israel: for $\varphi = 32°$, a gnomon of height 12 casts noon shadows of about $1\frac{3}{4}$, $7\frac{1}{2}$, and $17\frac{1}{2}$, at summer solstice, equinox, and winter solstice respectively. If, however, we go south to the Tropic of Cancer (taking the obliquity of the ecliptic to be 23° 40′), we obtain noon shadows of 0, $5\frac{1}{4}$, 13, and the approximation of 0, 6, 12 given in (1) becomes much more reasonable. Let us note that Ujjain, which was an ancient Indian astronomical center, has a latitude of 23° 11′. Furthermore, the exact noon shadow scheme (1) of BdS1 is found in the *Arthaśāstra* of KAUṬILYA[39] (Book 2, 20 : 41, 42) and in the *Vasiṣṭhasiddhānta*[40], as recorded in the *Pañcasiddhāntikā* II, 9. In the *Arthaśāstra* (20 : 10) it is explicitly stated that the height of the gnomon is 12 fingers. Thus, it seems probable that the gnomon procedure in BdS1 was transmitted from India.

E. Date and provenance of BdS1

It was mentioned above that the medieval Jewish writers attributed BdS1 to SHĔMUEL; let us examine to what extent this attribution is in accord with the talmudic sources[41]. In the talmud, SHĔMUEL is referred to[42] as YARḤINAAH ("lunar astronomer"); he states[43] "The paths of the heavens are as clear to me as the streets of Nĕharde'a". At a time when the new moon was proclaimed by observation, he was able to calculate a calendar in advance[44]. In 'Eruvin 56a, he uses the approximation of $365\frac{1}{4}$ days for the length of the year, the same length which is used in BdS1 (Chapters 4 and 5). More significantly, it is shown in the Appendix that the astrology of BdS1 is the same as SHĔMUEL's astrology as reflected in the talmud, both in scope and in details.

SHĔMUEL was a good friend of the Sasanian emperor SHAPUR I[45] (240–270). Persian sources recorded that SHAPUR encouraged the spread of Indian and Greek science within his empire[46]. This process was already under way in the time of his predecessor ARDASHIR I[47]. Thus, the plausible Indian origin of the shadow

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[39] KAUṬILYA, *Arthaśāstra*, ed. transl. & comm. R. P. KANGLE, 3 Vols., Bombay 1960–1965. D. PINGREE ("The Mesopotamian origin of early Indian mathematical astronomy", *Journal for the history of astronomy* iv (1973), 1–12, p. 1), states that it seems "fairly secure" that our recension of Book 2 was not composed before the 2nd century C.E.

[40] The original of which was probably composed in the 2nd or early 3rd century C.E. (PINGREE[39], p. 2).

[41] All talmudic references from here on are from the Babylonian Talmud, unless otherwise stated.

[42] Bava Meṣi'a 85b.

[43] Bĕrakhoth 58b.

[44] Rosh Hashanah 20b, Ḥullin 95b.

[45] Bĕrakhoth 56a, Sukkah 53a, Sanhedrin 98a.

[46] See D. PINGREE, "Astronomy and astrology in early India and Iran", *Isis* liv (1963), 229–246, p. 241, and the reference in his n. 96.

[47] See PINGREE[46], pp. 240–241 on *nakṣatra* astrology.

procedure in BdS1 causes no difficulty. On the contrary: a corollary to the Indian connection would be that BdS1 was most likely written in Babylonia rather than the Land of Israel, which was outside the Sasanian empire.

SARFATTI[48] already remarked that the cosmography and literary style of the first four chapters of BdS seem to indicate that they were composed during the talmudic period. In summary, there is no reason to doubt the medieval tradition that BdS1 is a Babylonian rabbinic product of the talmudic period, and more specifically, that SHĔMUEL was the author. It seems that at least some of the astronomy of BdS1 (*e.g.*, the shadow theory) originated in India; if part of its astronomy were non-Indian in origin, then it is possible that it came from a source which was also the source for the lunar theory of the calendar council in the Land of Israel. But if all the astronomy of BdS1 were derived from India, that would exclude any possible connection with the calendar council, because the talmudic lunar theory (to the extent that it existed) goes back to the first century B.C.E., as we shall see below in Section 5.

Thus the question of whether BdS1 could serve as a direct link with a lunar theory of the *sod ha'ibbur*, remains unsettled. In any case, BdS1 is instructive in that it gives us a glimpse of mathematical astronomy from the talmudic period.

4. Other midrashic parameters

In addition to the lunar visibility parameter of *Pirqey dR. Eli'ezer*, and the methods and parameters of BdS1, there are two interesting midrashic parameters, which we shall discuss in this Section.

A. Rising amplitudes

Baraitha Dĕ'arba'im Watesha'Middoth is a tannaitic work (2nd or early 3rd century C.E.) which is cited by the medieval rabbis; a long passage from it is preserved in the medieval midrashic anthology *Yalquṭ Shim'oni* (Pequdey 417–426). In *Yalquṭ* Pequdey 418 we read:

> He arranged them into standards[49] like the heavenly standards: the tribe of Judah in the east together with Issachar and Zebulun—corresponding to them in the heavens are Aries Taurus and Gemini with the sun in the east, they function (*mĕshammĕshin*) five out of eight parts in the east.

The key to understanding the significance of the ratio 5/8 lies in a talmudic discussion ('Eruvin 56a). A tannaitic source explains how to use the points of sunrise and sunset on the solstices in order to determine the cardinal directions. The wording might be construed as implying that on the solstices the sun's rising and setting amplitude is 45°; to negate this possibility, RAV MĔSHARASHIA quotes the tannaitic dictum:

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[48] SARFATTI[22], p. 76.

[49] The reference is to Numbers Ch. 2.

> The sun never rose from the northeast corner, and never set in the northwest corner; it never rose from the southeast corner, and never set in the southwest corner.

In other words, the sun's maximum amplitude is said to be recognizably less than 45°.

The *Yalquṭ's* ratio of 5 : 8 serves to quantify the above statement by saying that during the course of the year, the sun rises on an arc of the horizon which is only 5/8 of the eastern quarter[50] of the horizon. This would make the sun's solsticial amplitude $28\frac{1}{8}°$, which is reasonably accurate for Jerusalem in the time of the Second Temple[51].

In the *Yalquṭ* passage quoted above, the meaningful ratio of 5 : 8 is embedded into a scheme which assigns the zodiacal signs to the four directions (Aries Taurus Gemini—east; Cancer Leo Virgo—south, *etc.*), a scheme which has no astronomical significance. Such "mixtures" are common in antiquity, and this does not detract from the signficance of the ratio itself. It should be noted that the Jewish sages of the Temple period were keenly aware of the directions of the rising sun's rays on the solstices. The Talmud Yĕrushalmi (ʿEruvin 5 : 1), in explaining how the axis of the Temple was oriented exactly east-west, states: "The first prophets took great pains in making the Eastern Gate so that the sun should shine through it exactly, on the winter solstice and on the summer solstice."[51a]

B. Ecliptic limits

PdRE (Chapter 7) states:

> At the moment when the flame of the moon reaches the sun during the day, within sixty *maʿaloth*, it passes through it and darkens[52] its light; and at the moment when the flame of the sun reaches the moon during the night, within forty *maʿaloth*, it passes through it and darkens[52] its light.

This seems to be the remnant of a rule which stated that a solar eclipse will occur if the moon's latitude at conjunction is at most 60 units, and a lunar eclipse will take place if the lunar latitude at opposition is at most 40 units[53]. It is not clear what unit is meant by "*maʿaloth*", but the end result is that the ratio of latitudinal ecliptic limits for the sun and moon, respectively, is given as 3 : 2.

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[50] *i.e.*, from northeast to southeast.

[51] Jerusalem's latitude is $\varphi = 31° 47'$. Taking the obliquity of the ecliptic to be $\varepsilon = 23° 41'$, the exact formula $\sin a = \sin \varepsilon / \cos \varphi$ yields an amplitude of $a = 28.2°$.

[51a] In other words, at sunrise of the summer solstice, the sun's rays which passed through the Eastern Gate hit a certain distinguished point on a structure in the southern side of the Temple court, while on the winter solstice, they hit the corresponding symmetric point in the northern side.

[52] The mss. have *mkhh* "darkens", while the printed eds. have *mkbh* "extinguishes".

[53] At conjunction, the moon's latitude is also the distance between (the centers of) the sun and moon; at opposition, the lunar latitude measures the distance between the moon and the point diametrically opposed to the sun's center.

5. Evidence from the talmud; conclusion

Evidence of a rabbinical mathematical astronomy is not confined to the midrash. In the Talmud Shabbath (75a), RAV[54] and BAR QAPPARA[55] emphasize the importance of calculating "*tĕqufoth* and *mazzaloth*", *i.e.*, astronomical calculations. Rabbi YONATHAN[56] goes on to say:

> From where [do we know] that it is a commandment for one to calculate *tĕqufoth* and *mazzaloth*? For it is written (Deuteronomy 4 : 6) 'and you shall keep it and do it, for it is your wisdom and understanding in the eyes of the nations.' What is considered wisdom and understanding in the eyes of the nations? It must be calculation of *tĕqufoth* and *mazzaloth*.

The rabbis must have been referring to the calculation of phenomena based on true positions, rather than mean positions. Indeed, the local gentile astrologers all purported to calculate true positions, so that mean phenomena would hardly impress the gentiles.

For the particular question of talmudic lunar theory, our attention must center around Rabban GAMLIEL II, Prince of Israel (end of 1st, beginning of 2nd centuries C.E.). It was he who used images of lunar crescents in order to help him examine witnesses of the new moon[57]. In discussing the question of lunar visibility, he said to the Sages: "I received the following tradition from the House[58] of my father's father: sometimes [the moon] travels the long way, sometimes it travels the short way."[59] It was also from his father's father's House that he received the tradition concerning the length of the synodic month: "29 and a half days, two thirds of an hour, and 73 parts."[60]

By the literal meaning of his words, Rabban GAMLIEL's lunar theory must have gone back at least as far as his grandfather, Rabban GAMLIEL the Elder (first half of 1st century C.E.). It is likely that it originated with his great-great-grandfather, HILLEL the Elder, who emigrated from Babylonia to Jerusalem in the first century B.C.E., and founded the princely line. That HILLEL dealt with astronomy can be deduced from the following talmudic passage[61]:

> Hillel the Elder had eighty students... the least of them was Rabban YOHANAN BEN ZAKKAI. They said, concerning R. YOHANAN BEN ZAKKAI, that he did not forgo scripture, mishnah, talmud, ..., astronomy and mathematics (*tĕqufoth wĕgimatrioth*), ...

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54 Babylonia, first half of 3rd century C.E.

55 Land of Israel, end of 2nd—beginning of 3rd century C.E.

56 This is the reading in the manuscripts and medieval citations. (See R. N. RABINOVICZ, *Diqduqey Sofĕrim* (*Variae lectiones*), Tractate Shabbath, Munich 5635 (1875), *ad loc.* The printed editions have R. YOḤANAN). R. YONATHAN flourished in the Land of Israel in the first half of the 3rd century C.E.

57 Mishnah Rosh Hashanah 2 : 8 (24a).

58 *i.e.*, the princely dynasty.

59 Talmud Rosh Hashanah 25a.

60 *ibid.* (1080 parts = 1 hour).

61 Sukkah 28a, Bava Bathra 134a.

The implication[62] is that he learned all these things from his teacher HILLEL.

The parameter $\mu = 29^d\, 12^h\, 793^p$ given by Rabban GAMLIEL is, of course, the value of the mean synodic month in Babylonian System B[63]. Despite R. GAMLIEL's enigmatic way of presenting the parameter ("The renewal of the moon is not less than ..."), it must have been his mean, not minimum, value. This is so for the following reasons: (1) A lunar theory (even a rough one) based on μ as a *minimum* would not last for a month. (2) Starting with a lunar year of 354 days as a basis, RAVINA (end of 4th, beginning of 5th centuries C.E.) states[64] that the $\frac{2}{3}$ hour in μ adds up to a day in three years, and that the remainder adds up to a day after 30 years[65]; this makes sense only if μ is the mean. (3) Tradition, as reflected in the fixed Jewish calendar,[66] takes μ to be the mean synodical month.

In the beginning of this article, I showed that MAIMONIDES' procedure for determining lunar visibility seems to point to a Jewish, non-Arabic origin, going back to talmudic times. The combined weight of the midrashic and talmudic parameters, methods, and statements discussed above, tends to strenghthen this plausible possibility, perhaps even transforming it into a probability.

The talmud[67] mentions a tannaitic work called *Sod ha'ibbur* (secret of the calendar)[68]; the passage quoted from it deals with the question of lunar visibility. Now Rabbi LEVI (end of 3rd, beginning of 4th centuries C.E.) deduced[69] from a biblical verse that the Jewish people was adjured, among other things, not to reveal the *sod ha'ibbur*[70] to the gentiles. From the very plausible working hypothesis that the calendar council possessed a visibility theory which ultimately reached MAIMONIDES, the above adjuration acquires its full significance: for the greater part of the millennium following the death of Babylonian astronomy in the first century C.E., Israel would be the only nation possessing a serious lunar visibility theory[71].

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62 See also Tractate Soferim 16: 8, 9 (16: 6, 7).

63 The earliest appearance of function *G* of System B (which has μ as its mean value) is in an ephemeride dated 208 B.C.E. (NEUGEBAUER, *Astronomical cuneiform texts*[11], text 170, p. 172).

64 Talmud 'Arakhin 9b.

65 "year" here must refer to a twelve-month period, because an intercalatory year would destroy the very basis of 354 days upon which RAVINA is working.

66 Rav HAI GAON (939–1038) and others write that HILLEL II established the fixed calendar in the year 358/359 C.E. For references and discussion, see M. M. KASHER, *Torah shelemah*, Vol. **13**, New York 5710 (1949), pp. 24–37.

67 Rosh Hashanah 20b.

68 In this context, "*sod*" simply means "secret", in contrast to its use in Kěthubboth 112a, where it means "secret council". *Cf.* note 15, and MAIMONIDES 11:4.

69 Kěthubboth 111a.

70 "*Sod ha'ibbur*" (Kěthubboth 111a) is the reading in eight manuscripts, the *Yalquṭ*, and some medieval citations. Some other manuscripts, Rashi, and the printed editions, have only "*sod*". See *The Babylonian Talmud*, Tractate Kethubboth II, Institute for the Complete Israeli Talmud, Jerusalem 5737 (1977), *ad loc.*

71 By a serious visibility criterion, I mean a criterion which depends, continuously, on the two variables: elongation and arc of vision (see note 8a), or on two equivalent variables; (for example, the sun's depression below the horizon at moonset may be sub-

Appendix:

Shĕmuel's Astrology in the Talmud, and the Astrology of BdS1

In the talmud (Shabbath 156a), Rabbi Ḥanina[72] lists the influences which the various planets have on a person's life, according to the planetary hour in which he was born. He then makes a general statement: "*Mazzal*[73] makes wise, *mazzal* makes rich, and there is *mazzal* for Israel", *i.e.*, the stars determine the destiny, in this world[74], of each individual Jew. The talmud goes on to list various sages, including Shĕmuel, who differ with Rabbi Ḥanina and hold that "there is no *mazzal* for Israel". It is related (*ibid.* 156b) that Shĕmuel was sitting with Ableṭ, a gentile scholar and astrologer, while some men were on their way to cut reeds. Ableṭ said to Shĕmuel, "This man will not come back; he will be bitten by a snake and die." Shĕmuel replied: "If he is an Israelite, he will come back."

Shĕmuel, then, denied the validity of genethlialogical astrology for Jews. His astrological comments in the talmud are limited to prognostications of weather: the effect of Jupiter on the weather[75], and the influence of the constellations Kĕsil and Kimah on the temperature[76],[77]. This is perfectly in line with BdS1, whose astrology deals only with prognostications of weather (Chapter 4). This agreement extends to the details of Shĕmuel's astrology. In 'Eruvin 56a, we read:

> Shĕmuel said: A vernal equinox which falls in[78] Jupiter will surely cause the trees to break, and a winter solstice which falls in Jupiter will surely dry out the seedlings „provided that the [previous] new moon took place in the moon or in Jupiter.

We see, then, four elements of Shĕmuel's astrology: (1) A planet exerts its influence not by its position in the ecliptic, but rather through the hour with which it is associated. (This is true of all talmudic astrology.) (2) The planet on which a *tĕqufah* (equinox or solstice) falls, influences the following season's weather. (3) An additional factor is the planet on which the previous lunar conjunction falls. (4) The specific effects of Jupiter mentioned above (effects brought about by wind).

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stituted for the arc of vision). In pre-Islamic Indian astronomy, the new moon is said to be visible if and only if the arc of vision $\geqq 12°$ (see, *e.g.*, the *Pañcasiddhāntikā*[36] V, 1–3), so that the Indian criterion fails to make the grade.

[72] A contemporary of Shĕmuel who emigrated from Babylonia to the Land of Israel.

[73] *Mazzal* is a general term which includes luminaries, planets, and stars.

[74] In contradistinction to the World to Come; see the discussion in Ta'anith 28a concerning R. El'azar b. Pĕdath.

[75] 'Eruvin 56a, quoted below *in extenso*.

[76] Bĕrakhoth 58b.

[77] In Shabbath 129b, after giving certain medical advice concerning bloodletting, Shĕmuel states that it should be done on Sunday, Wednesday, or Friday. The talmud assumes that Tuesday is excluded for astrological reasons (Mars is in an even hour during the day.) But here too, there is no dependence on genethlialogical data; it is rather like the weather, which affects everyone equally.

[78] *i.e.*, in the hour of Jupiter; similarly in the sequel.

In BdS1, we find the same four elements: (1) is equally true for BdS1. The beginning of Chapter 4 is devoted to an exposition of (2) for each of the seven planets, with the following formula: If the *tĕqufah* falls on planet x, in the hot season the result is y, in the cold season z. BdS1 goes on to say that if the *tĕqufah* and the new moon are both "in one way", the effect of the planetary hour gets reinforced. This is in line with element (3), where SHĔMUEL says, in effect, that if both *tĕqufah* and new moon fall in the hour of Jupiter, the outcome is severe[79]. As for Jupiter's effect (4), BdS1 states "Jupiter is the planet of wind" and

> If the *tĕqufah* falls ... in the planet of wind, in the hot season "stormy wind which carries out His word" (Psalms 148: 8), in the cold season "a great and strong wind" (Kings I, 19 : 11).

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[79] Another example along the lines of (3) is found further on in Chapter 4, where BdS1 lists the combinations of zodiacal sign and planetary hour (Cancer-sun, Virgo-Venus, *etc.*) in which the moon causes rain. Here "the moon" means new moon, or perhaps full moon.